Inference for Means

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Math 141, 4/19/21

Outline

In this lecture, we will...

- Investigate the t distribution.
- Create confidence intervals and perform hypothesis tests using t distribution for sample means.

Section 1

The t-distribution

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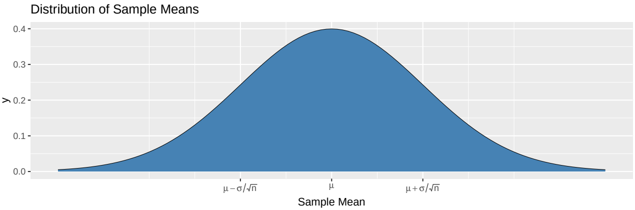

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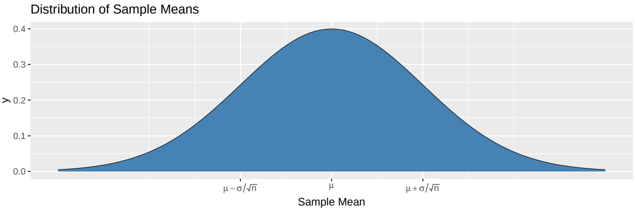

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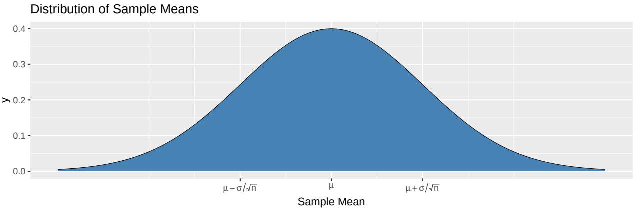

- Note that smaller σ and larger n both correspond to smaller standard error.
- As n increases, Normal approximation becomes more accurate, even if population is skewed.

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  no longer follows a standard Normal distribution!
- Instead, the standardized statistic follows the t-distribution
    - The t-distribution was first studied in 1908 by Willam Gosset, who published under the pseudonym *Student*.

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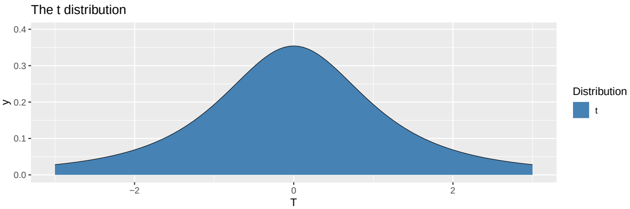

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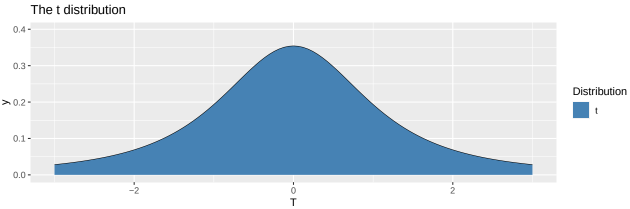

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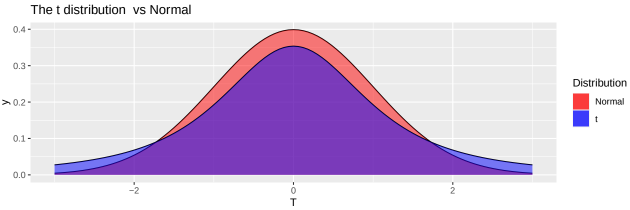

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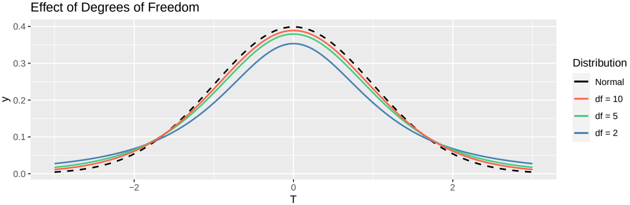

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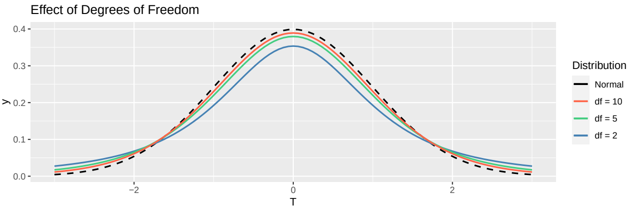

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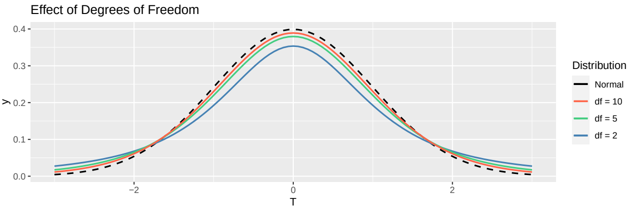

- As degrees of freedom increases, the t distribution gets closer to the Normal distribution.
  - For $df \geq 30$, the t distribution is nearly indistinguishable from the Normal

Distribution of Sample Means using t-distribution

Theorem

Suppose a sample of size n is collected from a population with mean μ. The distribution of the sample mean $\bar{x}$ has the following characteristics:

- ***Center**: The mean is equal to μ*
- ***Spread**: The standard error is equal to $\frac{s}{\sqrt{n}}$ (where s is the sample st. dev.)*
- ***Shape**: The standardized statistic follows approximately a t-distribution with $n-1$ degrees of freedom.*

For small sample sizes ($n \leq 30$), the t-distribution is only a good approximation if the population distribution is approximately Normal.

Section 2

Statistical Inference

The Origin Story

A batch of stout beer is best when it has an *original gravity* (OG) close to 1.071. The particular OG of a batch depends on a number factors (like temperature, rest time, recipe, etc.).

If we can only obtain a small number of measurements from the batch, how can we quantify whether the deviations we observe are due to random sampling, and not an actual deviation in OG?

Confidence Intervals

The t-procedures for Confidence Intervals

A $C\%$ confidence interval for a population mean μ using a sample of size n is

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where $\bar{x}$ and s are the mean and standard deviation of the sample, and where t^* is the critical value for $C\%$ confidence in the t-distribution with $n-1$ degrees of freedom.

The t-procedures are appropriate if $n \leq 30$ and the population is approximately Normal, or if $n > 30$.

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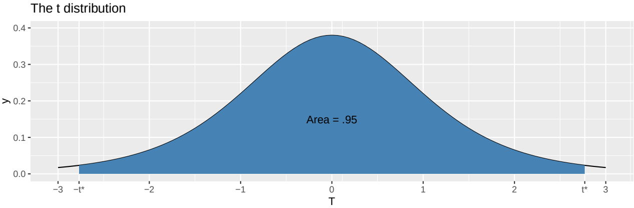

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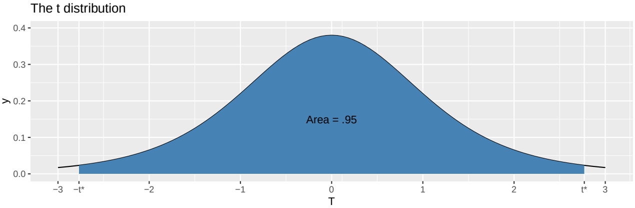

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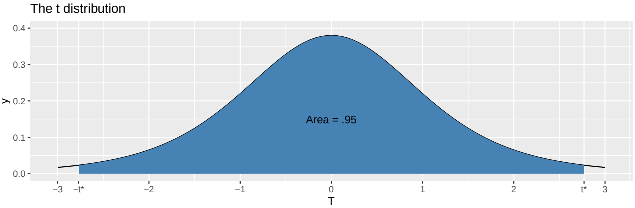

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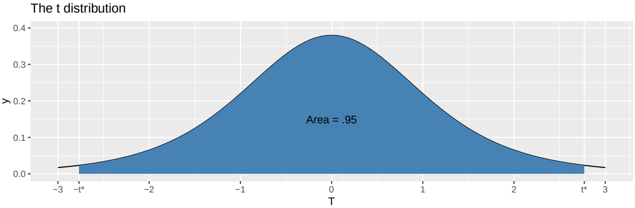

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- Note that the $t*$ critical value of 95% confidence is larger than the z^* critical value

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- Since $\mu = 1.071$ is within this interval, it is plausible that the batch has the desired OG.

Comparison using infer

If we instead use infer...

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set.seed(1908)
boot_beer <- beer %>%
  specify(response = OG) %>%
  generate(reps = 5000, type = "bootstrap" ) %>%
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ci_beer <- boot_beer %>%
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- The bootstrap interval is a bit narrower than the theory-based interval:

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Hypothesis Tests

The t-test for Single Mean

To test $H_0 : \mu = \mu_0$ against $H_a : \mu \neq \mu_0$ (or 1-sided alternatives), use the t-statistic

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

where $\bar{x}$ and s are the mean and standard deviation of the sample with size n. The distribution of t is approximated by the t-distribution with $n - 1$ degrees of freedom.

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- Therefore, our t-statistic is

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{1.0686 - 1.071}{\frac{0.0063}{\sqrt{5}}} = -0.845$$

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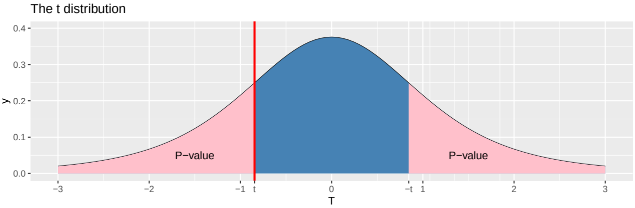

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P_value<-2*pt( q = -.845 , df = 4)
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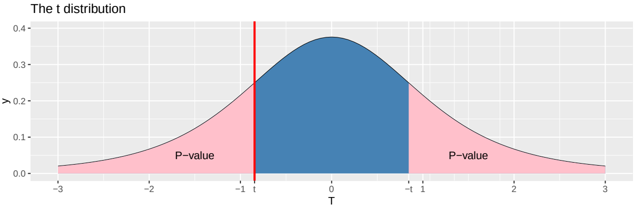

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- Our sample is consistent with a true mean OG of $\mu = 1.071$.

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pval_beer <- null_beer %>%
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- The bootstrap p-value is a bit larger than the theory-based p-value:

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  - $n \geq 60$, and the population is not extremely skewed
- In general, for small sample sizes, neither method should be used if population does not appear Normal. But if it is Normal, theory-based methods will be more accurate.
- For moderate sample sizes with moderate skew, simulation-based methods will be more accurate

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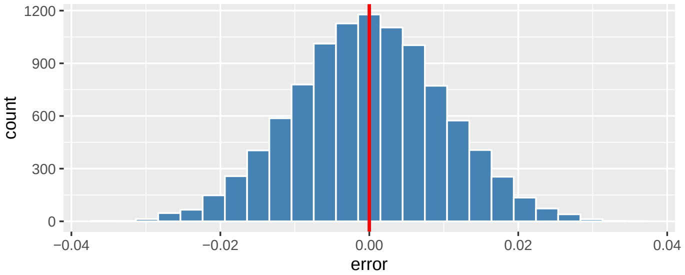
- To verify, we'll create 1000 95% confidence intervals using (a) the t-distribution and (b) the Normal distribution, and see how many contain the true population mean.
- Suppose we have the following population distribution for measurement errors
    - The mean error is 0.



1000 Samples

The following code collects 10000 samples from the population, each of size 4. It then computes the mean and standard deviation of each sample.

```
set.seed(1023)
samps<-population %>%
  rep_sample_n(size = 4, reps = 10000) %>%
  group_by(replicate) %>%
  summarize(avg = mean(error), st_dev = sd(error))
```

```
## # A tibble: 6 x 3
##   replicate      avg  st_dev
##       <int>    <dbl>   <dbl>
## 1         1 -0.00638 0.00580
## 2         2 -0.00110 0.00809
## 3         3  0.00312 0.0109
## 4         4 -0.00154 0.0130
## 5         5 -0.00337 0.00828
## 6         6 -0.00186 0.00640
```

The Confidence Intervals

- The critical value for a 95% confidence interval using...
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  - the t distribution with 3 df is $t^* = 3.18$.
- The following code creates confidence intervals for each sample:

```
samps <- samps %>% mutate(
  lower_z = avg - 1.96*st_dev/2, upper_z = avg + 1.96*st_dev/2,
  lower_t = avg - 3.18*st_dev/2, upper_t = avg + 3.18*st_dev/2)
```

```
## # A tibble: 6 x 7
##   replicate      avg  st_dev  lower_z   upper_z lower_t upper_t
##       <int>    <dbl>   <dbl>    <dbl>     <dbl>   <dbl>   <dbl>
## 1         1 -0.00638 0.00580 -0.0121  -0.000692 -0.0156 0.00285
## 2         2 -0.00110 0.00809 -0.00903  0.00683  -0.0140 0.0118
## 3         3  0.00312 0.0109  -0.00759  0.0138   -0.0143 0.0205
## 4         4 -0.00154 0.0130  -0.0143   0.0112   -0.0222 0.0191
## 5         5 -0.00337 0.00828 -0.0115   0.00474  -0.0165 0.00979
## 6         6 -0.00186 0.00640 -0.00813  0.00441  -0.0120 0.00831
```

Which intervals contain the true mean?

- Since we **know** the population has mean 0, we can determine whether each interval contains the true mean.

```
samps<-samps %>% mutate(
  z_success = ifelse(( lower_z < 0 & upper_z > 0 ) , "yes", "no"),
  t_success = ifelse(( lower_t < 0 & upper_t > 0 ) , "yes", "no"))
```

```
## # A tibble: 6 x 7
##   replicate  lower_z   upper_z lower_t upper_t z_success t_success
##       <int>    <dbl>     <dbl>   <dbl>   <dbl> <chr>     <chr>
## 1         1 -0.0121  -0.000692 -0.0156 0.00285 no        yes
## 2         2 -0.00903  0.00683  -0.0140 0.0118  yes       yes
## 3         3 -0.00759  0.0138   -0.0143 0.0205  yes       yes
## 4         4 -0.0143   0.0112   -0.0222 0.0191  yes       yes
## 5         5 -0.0115   0.00474  -0.0165 0.00979 yes       yes
## 6         6 -0.00813  0.00441  -0.0120 0.00831 yes       yes
```

Which intervals contain the true mean?

- Since we **know** the population has mean 0, we can determine whether each interval contains the true mean.

```
samps<-samps %>% mutate(
  z_success = ifelse(( lower_z < 0 & upper_z > 0 ) , "yes", "no"),
  t_success = ifelse(( lower_t < 0 & upper_t > 0 ) , "yes", "no"))
```

```
## # A tibble: 6 x 7
##   replicate  lower_z   upper_z lower_t upper_t z_success t_success
##       <int>    <dbl>     <dbl>   <dbl>   <dbl> <chr>     <chr>
## 1         1 -0.0121  -0.000692 -0.0156 0.00285 no        yes
## 2         2 -0.00903  0.00683  -0.0140 0.0118  yes       yes
## 3         3 -0.00759  0.0138   -0.0143 0.0205  yes       yes
## 4         4 -0.0143   0.0112   -0.0222 0.0191  yes       yes
## 5         5 -0.0115   0.00474  -0.0165 0.00979 yes       yes
## 6         6 -0.00813  0.00441  -0.0120 0.00831 yes       yes
```

- What proportion of z- and t-intervals contain 0?

```
## # A tibble: 1 x 2
##   z_rate t_rate
##    <dbl>  <dbl>
## 1  0.859  0.953
```