Homework 10

Insert Name

Math 141, Week 10

Due: 11:59pm, Friday April 8

Instructions

Work through the problems below and submit this document as a knitted .pdf to the Math 141 S22 Wells Lecture gradescope page.

For each problem, put your solution between the bars of red stars.

Acknowledgements

If you work with a classmate, please write a note acknowledging this.

Exercise 1

Suppose A and B are events with P(A) = 0.3 and P(B) = 0.7.

a. Is it always possible to compute P(A and B) if you only know P(A) and P(B)?

b. Assuming that events A and B arise from independent random processes, find P(A and B) and P(A | B).

c. Suppose we are given that P(A and B) = 0.1. Are events A and B independent? Explain.

d. If we are given that P(A and B) = 0.1 , what is P(A | B)?

Exercise 2

Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease. The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease. There is a line from the Fox television show *House*

that is often used after a patient tests positive for lupus: "It's never lupus." Do you think there is truth to this statement? Use appropriate probabilities to support your answer.

Exercise 3

Earlier in the term, students in Math 141 were asked two questions as part of a larger survey: "Are hot dogs sandwiches?" and "If dogs wore pants, would they wear them on their front legs, back legs, or on all four legs"? A summary of responses for 70 students is given below:

	Hotdog is a sandwich	Hotdog is not a sandwich
Front legs	1	1
Back legs	17	26
All 4 legs	9	16

Suppose we randomly choose 1 student who completed this survey.

a. Are the events "the student thinks a hotdog is not a sandwich" and "the student thinks dogs should wear pants on their back legs" mutually exclusive?

b. What is the probability that the randomly chosen student thinks a hotdog is a sandwich?

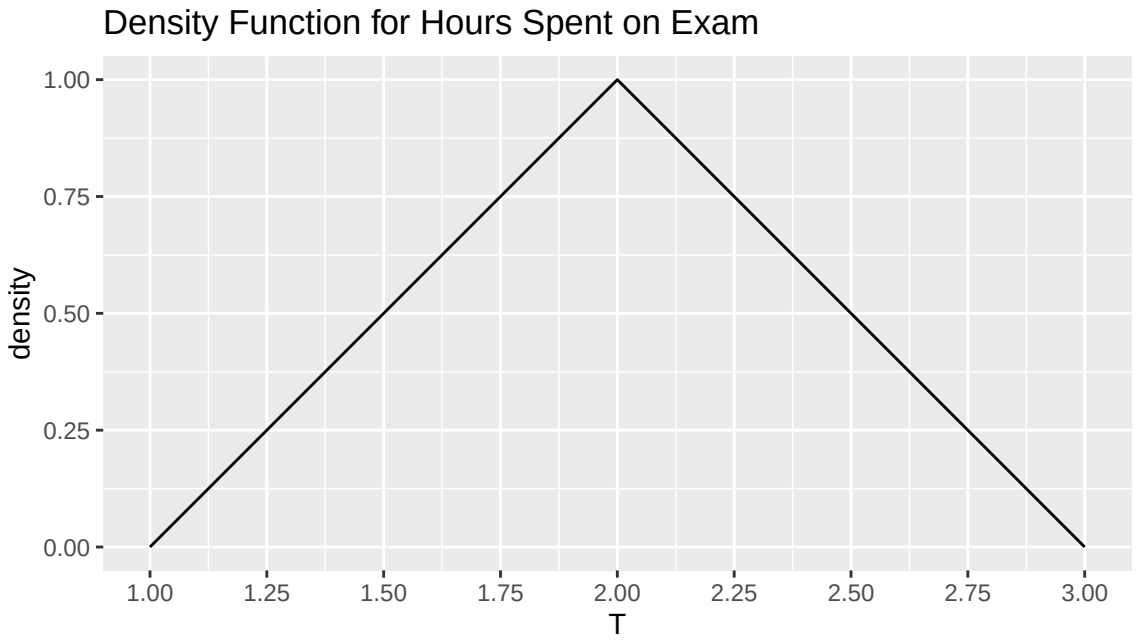
c. What is the probability that the randomly chosen student thinks a hotdog is a sandwich and thinks that dogs should wear pants on all 4 legs?

d. What is the probability that the randomly chosen student thinks a hotdog is a sandwich given that the student thinks dogs should wear pants on all 4 legs?

e. Is the event "the student things a hotdog is a sandwich" independent of the event "the student thinks dogs should wear pants on their front legs"?

Exercise 4

Suppose the number of hours a particular student takes to complete a statistics exam is modeled by a continuous random variable T with the following density function:



Models of this type arise frequently in settings where we have sparse information to model a continuous phenomenon (i.e. we may only have estimates for the minimum, maximum, and most likely values for the phenomenon).

For each of the following parts of the problem, use *geometric reasoning* to compute areas (you do not need to use calculus to answer any of these problems). Recall that the area of a triangle is $A = 0.5 * base * height$.

a. Verify that the function plotted above indeed has area of 1.

b. What is the probability that the student takes at most 2.5 hours to complete the exam?

c. What is the value of t so that a student has probability 0.08 of completing the exam before time t.

d. During what interval of length 1 does the student have the largest probability of completing the exam. That is, find a number a so that $P(a < T < a + 1)$ is as large as possible.