Correlation and Linear Models

Nate Wells

Math 141, 2/14/22

Outline

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- Discuss the relationship between correlation and causation
- Compare and contrast observational studies and random experiments
- Introduce linear models

Section 1

Assessing Relationships Between Variables

Explanatory and Response Variables

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  1. **Observational studies**, where researchers do not interfere with how data arises.
  2. **Random experiment**, where individuals are assigned to group and a random treatment is assigned.
- Usually, only random experiments may allow researchers to conclude a causal link between explanatory and response variables.

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- If variables X and Y are correlated, there are 4 possible explanations:
  1. Changes in X cause changes in Y
  2. Changes in Y cause changes in X
  3. Changes in a third variable Z cause changes in *both* X and Y
  4. The observed correlation in X and Y is due to chance.

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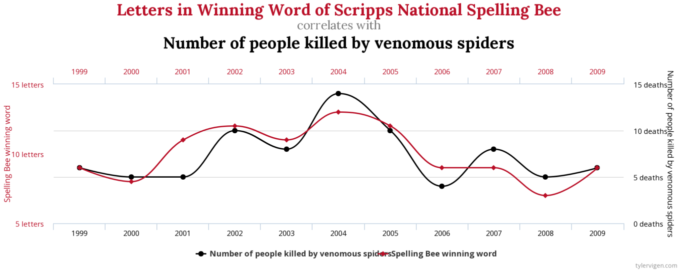

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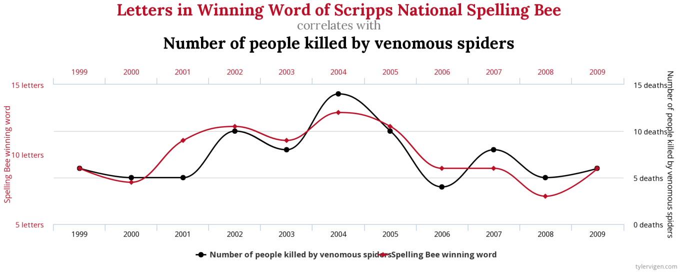

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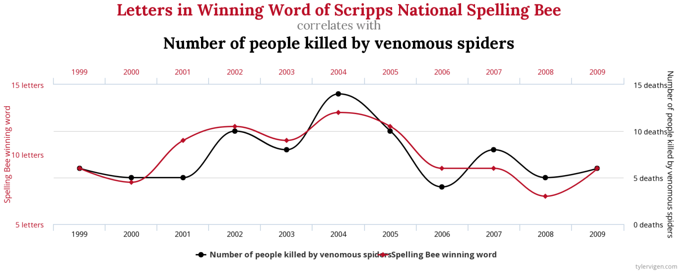

- How do we rule out spurious correlations?
  - Gather more data. If the correlation occurred by chance just due to sampling, the relationship is unlikely to be repeated in an independent sample.

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- How do we rule out confounding variables?
  - Create models that include possible confounding variables
  - Design experiments that control for possible confounding variables

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- How do you rule out reverse causation?
  - Investigate the temporal order of events.
  - Design an experiment where theorized cause is administered as treatment.

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  "If, for example, it were possible to infer that smoking cigarettes is a cause of this disease, it would equally be possible to infer on exactly similar grounds that inhaling cigarette smoke was a practice of considerable prophylactic value in preventing the disease, for the practice of inhaling is rarer among patients with cancer of the lung than with others."

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- Fisher did not disagree with the statistical analysis that smoking and cancer were highly correlated.
- So how do we know that Fisher was wrong? (He was)

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  2. **Consistency** Different studies should show similar effect.
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  4. **Temporality** The effect should occur before the cause.
  5. **Gradient** Greater exposure to cause should correspond to greater size of effect
  6. **Plausibility** A plausible mechanism should exist linking cause and effect.
  7. **Coherence** A cause and effect relationship should not conflict with other known relationships
  8. **Experimental Evidence** A cause and effect relationship should be evident in randomized experiment.
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  - No. But they are good guidelines.

Section 2

Experiments and Observational Studies

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  4. **Blocking**: If variables are suspected to affect response variable, subjects are first grouped into blocks based on these variables.

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- It is suspected that nitrate supplements may effect professional and amateur athletes differently, and so subjects are blocked for pro status:
  1. Divide SRS into pro and amateur blocks.
  2. Randomly assign pro athletes to treatment and control groups.
  3. Similarly, randomly assign amateur athletes to treatment and control groups.
  4. Ensure pro/amateur status is equally represented in treatment and control groups.

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- Experiments may not be manufacturable
  - To study whether high unemployment rate leads to presidential losses for the incumbent party, we cannot create new presidential races.
- Experiments of appropriate size may be prohibitively expensive
  - Experiments of small or moderate size often include uncontrolled confounding variables

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ideal experiment	Random assignment	No random assignment	most observational studies
Random sampling	causal and generalizable	not causal, but generalizable	Generalizability
No random sampling	causal, but not generalizable	neither causal nor generalizable	No generalizability
most experiments	Causation	Association	bad observational studies

Section 3

Introduction to Linear Regression

Overview

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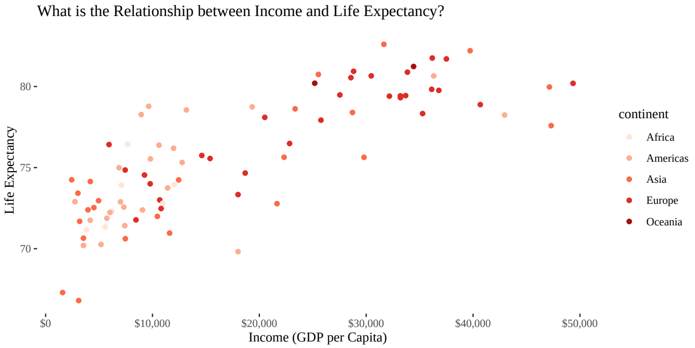

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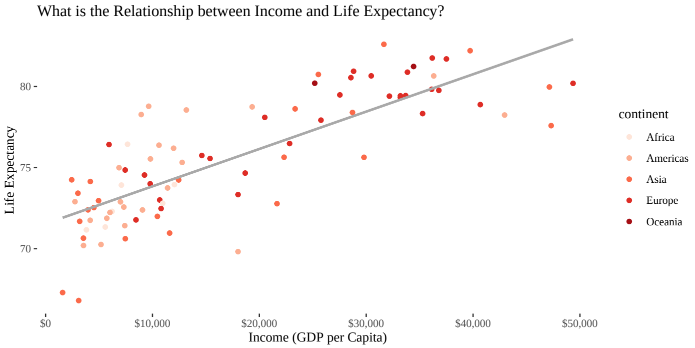

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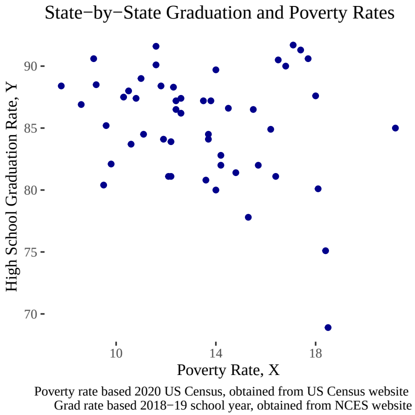
$$Y = \beta_0 + \beta_1 X \qquad \text{with } \beta_0, \beta_1 \text{ fixed constants}$$

- Of course, in the wild, the observed values of Y will **not** be perfectly predicted by the values of X.

$$Y = \beta_0 + \beta_1 X + \epsilon \qquad \text{where } \epsilon \text{ is the error}$$

Scatterplots and Linear Relationships I



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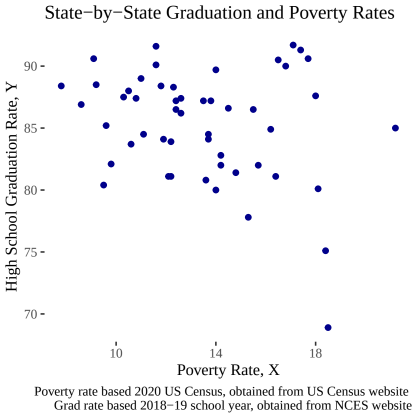

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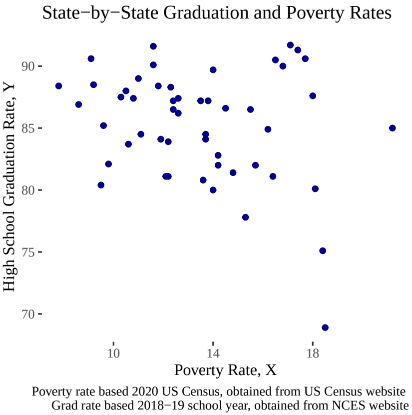

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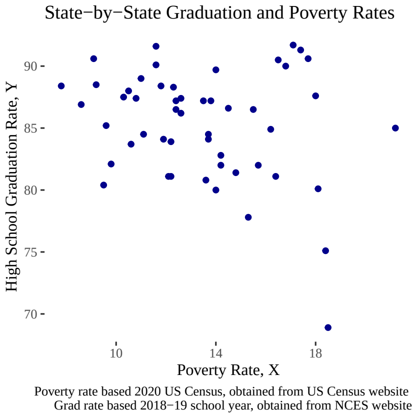

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- Response Variable:

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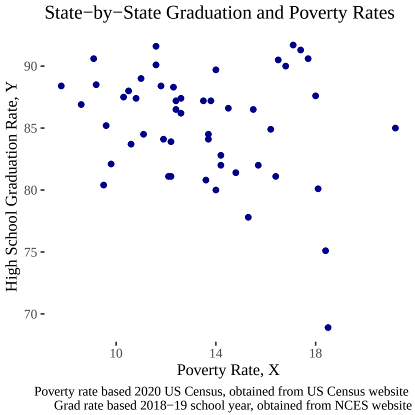
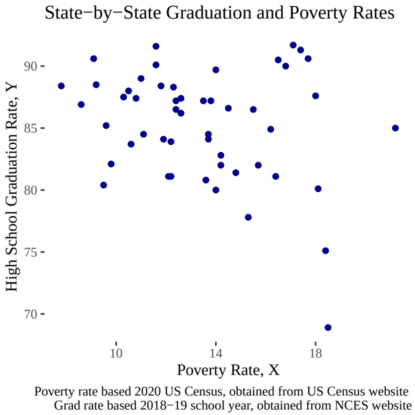


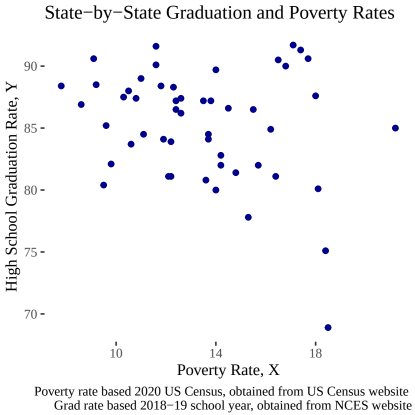
- Explanatory Variable:
  - Poverty Rate (X)
- Response Variable:
  - High School Graduation Rate (Y)

Scatterplots and Linear Relationships I



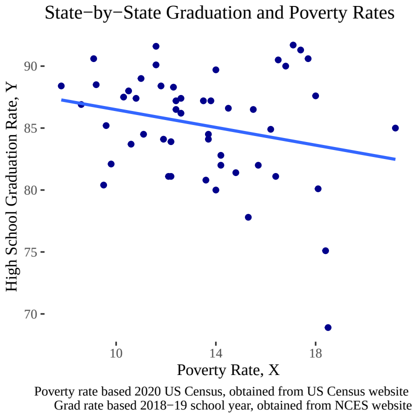
- Explanatory Variable:
  - Poverty Rate (X)
- Response Variable:
  - High School Graduation Rate (Y)
- Relationship:

Scatterplots and Linear Relationships I



- Explanatory Variable:
  - Poverty Rate (X)
- Response Variable:
  - High School Graduation Rate (Y)
- Relationship:
  - Linear, negative, moderately strong

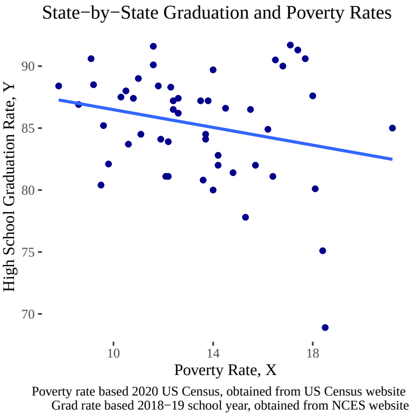
Scatterplots and Linear Relationships II



- Model (hand-fitted):

$$\hat{Y} = \beta_0 + \beta_1 X = 90 - 0.4X$$

Scatterplots and Linear Relationships II



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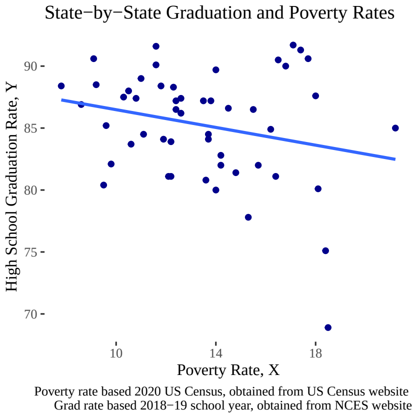
Scatterplots and Linear Relationships II



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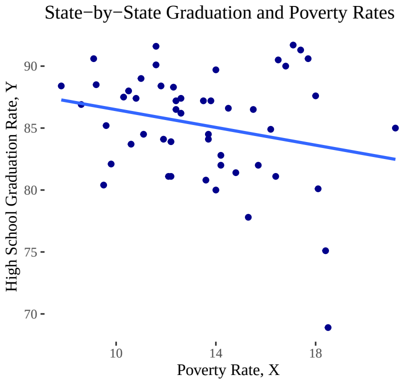
Scatterplots and Linear Relationships II



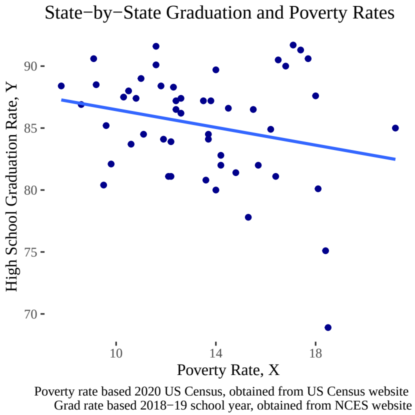
Poverty rate based 2020 US Census, obtained from US Census website
Grad rate based 2018−19 school year, obtained from NCES website

- Model (hand-fitted):

$$\hat{Y} = \beta_0 + \beta_1 X = 90 - 0.4X$$

- Hat ($\hat{Y}$) indicates this is an estimate of Y
- **Slope** of $\beta_1 = -0.4$ means every 1 unit increase in Poverty corresponds to a 0.4 unit decrease on average in Graduation.
- **Intercept** of $\beta_0 = 90$ means model predicts graduation rate of 90% when poverty rate is 0%.

Scatterplots and Linear Relationships III

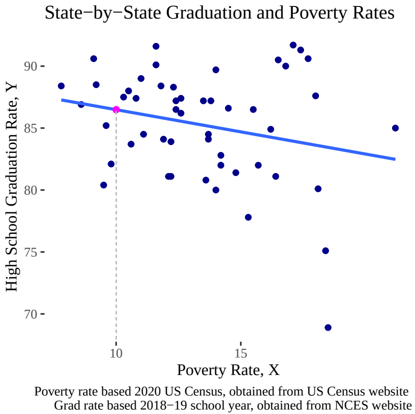


- Model:

$$\hat{Y} = 90 - 0.4 \cdot X$$

- What does the model predict to be the graduation rate for a state with theoretical poverty rate 10%?

Scatterplots and Linear Relationships III



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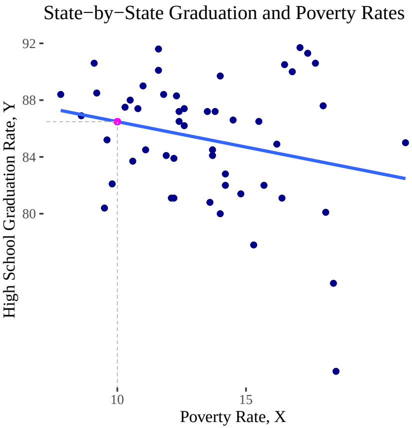
Scatterplots and Linear Relationships III



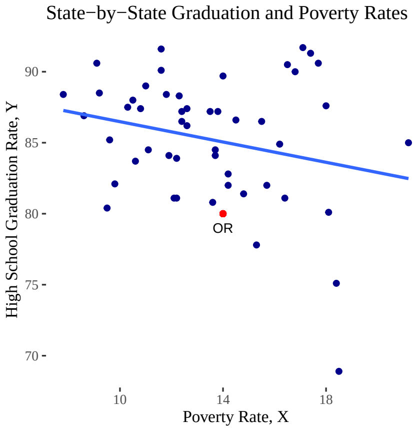
- Model:

$$\hat{Y} = 90 - 0.4 \cdot X$$

- What does the model predict to be the graduation rate for a state with theoretical poverty rate 7%?

$$\hat{Y} = 90 - 0.4 \cdot 10 = 86$$

Scatterplots and Linear Relationships IV



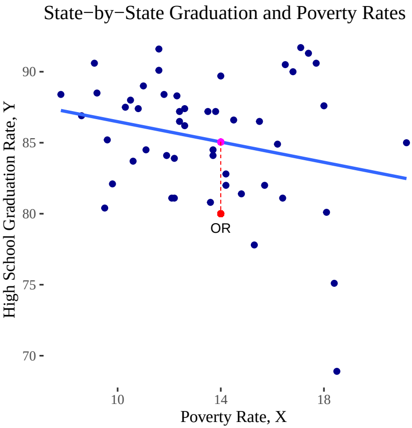
- Model:

$$\hat{Y} = 90 - 0.4 \cdot X$$

- Oregon has a poverty rate of 14. What does the model predict is Oregon's graduation rate?

$$\hat{Y} = 90 - 0.4 \cdot 14 = 84.4$$

Scatterplots and Linear Relationships IV



- Model:

$$\hat{Y} = 90 - 0.4 \cdot X$$

- Oregon has a poverty rate of 14. What does the model predict is Oregon's graduation rate?

$$\hat{Y} = 90 - 0.4 \cdot 14 = 84.4$$

But Oregon's actual graduation rate is 80

Residuals

- **Residuals** are the leftover variation in the data after accounting for model fit.
- Each observation (X_i, Y_i) has its own residual e_i, which is the difference between the observed (Y_i) and predicted $(\hat{Y}_i)$ value:

$$e_i = Y_i - \hat{Y}_i$$

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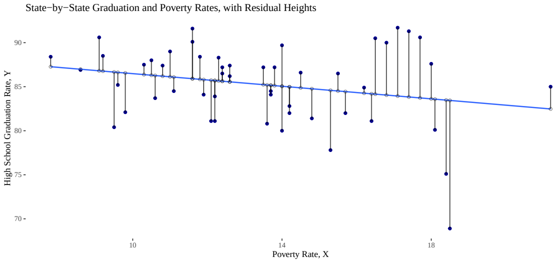

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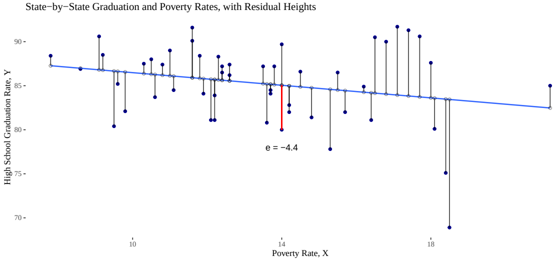
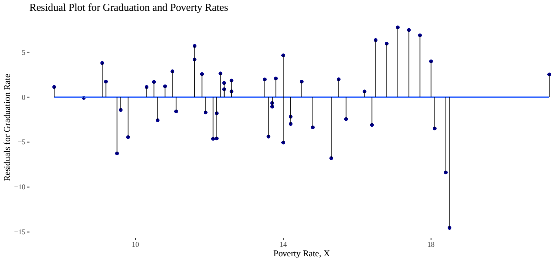


-Oregon's residual is

$$e = Y - \hat{Y} = 80 - 84.4 = -4.4$$

Residual Plot

- To visualize the degree of accuracy of a linear model, we use residual plots:



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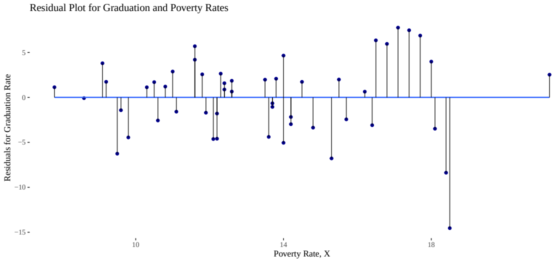

- Points preserve original x-position, but with y-position equal to residual.

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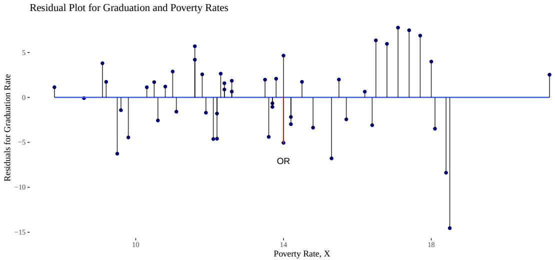

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