Confidence Intervals II

Nate Wells

Math 141, 3/11/21

Outline

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- Use bootstrapping as means of creating confidence intervals
- Interpret confidence intervals
- Implement the `infer` package to automate bootstrapped confidence intervals

Section 1

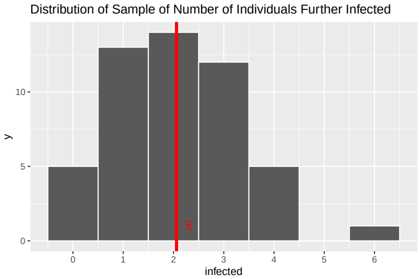
Bootstrapping Confidence Intervals

Reproduction Rate for Covid-19

Researchers are interested in the COVID-19 **reproduction rate** (the average number of individuals each infected person further infects)

- We have a sample of 50 infected individuals and perform contract tracing to determine how many other individuals each infects.

```
##   infected  n
## 1        0  5
## 2        1 13
## 3        2 14
## 4        3 12
## 5        4  5
## 6        6  1
##   mean_infected
## 1          2.06
```



- Goal: Create an interval of plausible values for the reproduction rate.

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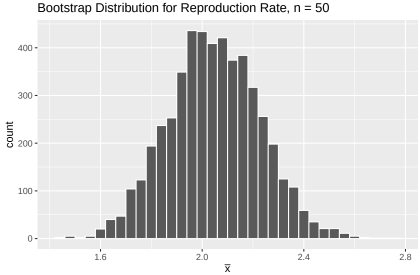

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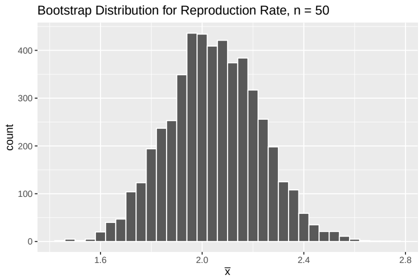

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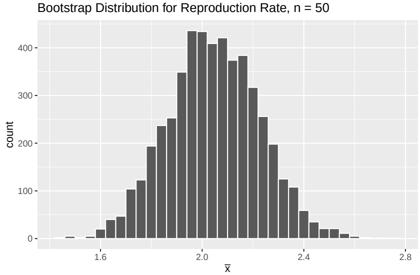

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5. The 95% confidence interval is

$$\bar{x} \pm 2 \cdot SE \qquad 2.06 \pm 2 \cdot 0.181$$

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General Confidence Intervals

The $C\%$ confidence interval for a parameter is an interval estimate that is computed from sample data by a method that captures the parameter for $C\%$ of all samples.

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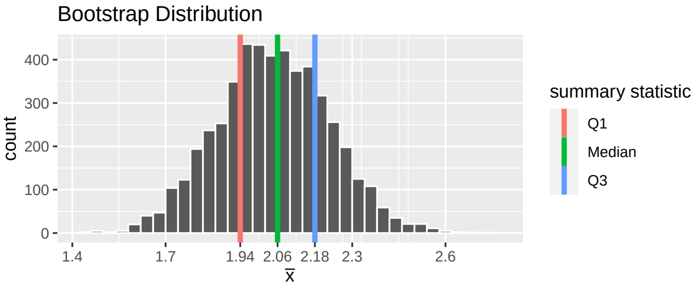

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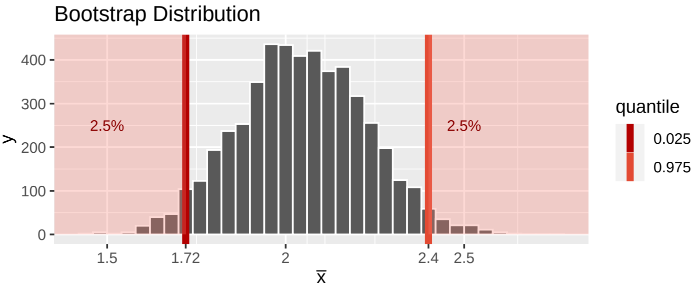

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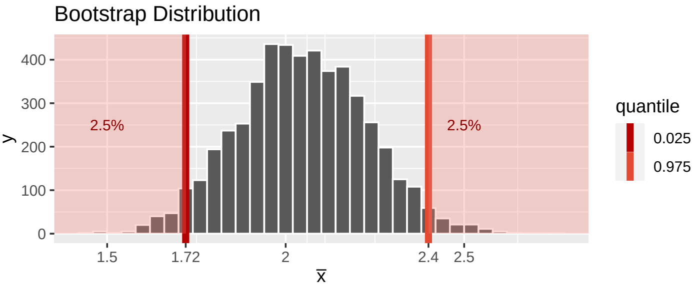

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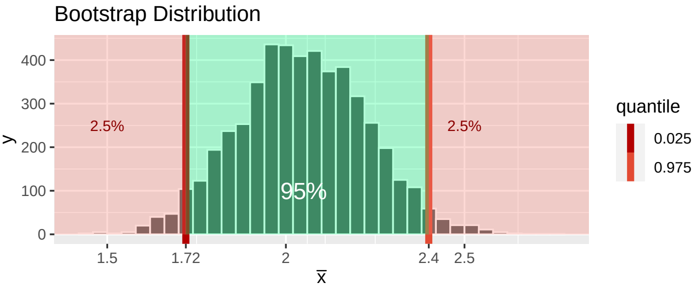

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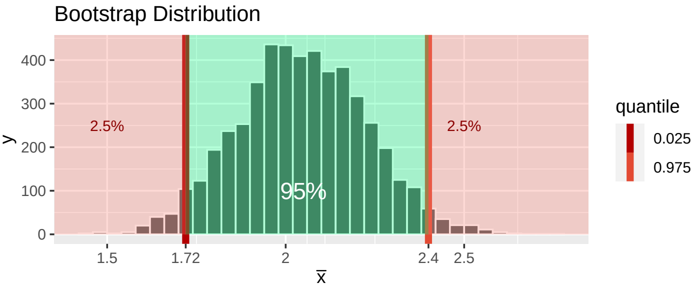

- But this means that 95% of the data is between the .025 and the .975 quantiles
  - For a sampling distribution that is approximately bell-shaped, the .025 quantile is about $2 \cdot SE$ below the mean, and the .975 quantile is about $2 \cdot SE$ above the mean

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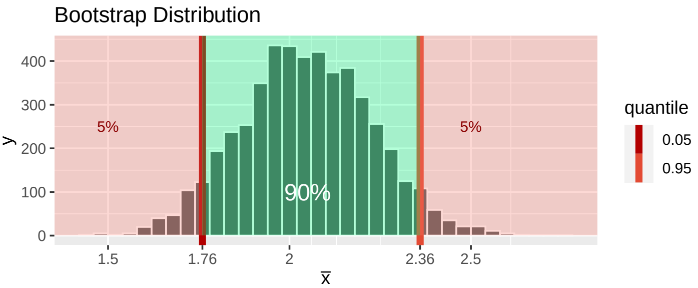

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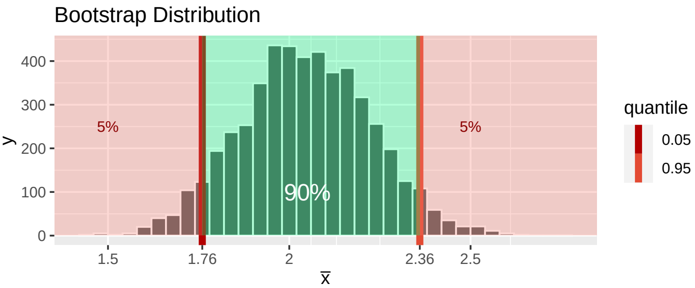

- We can use the `quantile` function in R to calculate the .05 and .95 quantiles

```
quantile(bootstrap_stats$x_bar, c(.05, .95))
```

```
##   5%  95%
## 1.76 2.36
```

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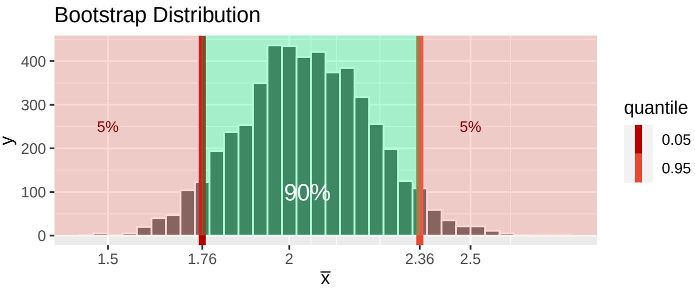

- Our 90% confidence interval is therefore 1.76 to 2.36

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- Increase sample size.
  - The standard deviation of the sampling distribution decreases as sample size increases. More sample means are closer to the true parameter
- Decrease confidence level.
  - The margin of error is determined by the percentiles. A 95% confidence interval is formed by the 2.5th and 97.5th percentiles in the bootstrap distribution.
  - Decreasing confidence level brings the percentiles closer to the 50th percentile, decreasing the width of the interval.

Section 2

Confidence Interval Misunderstandings

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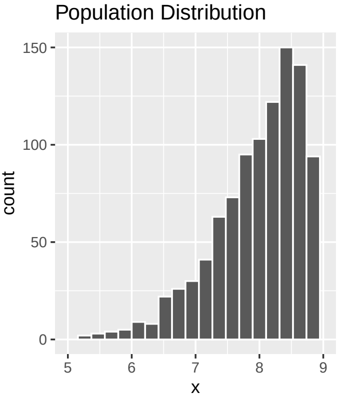

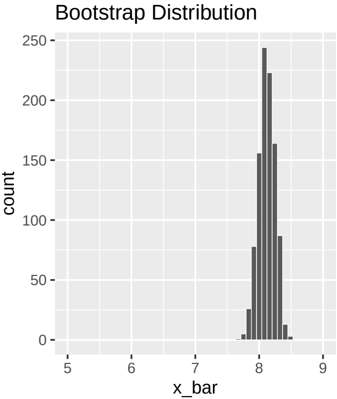

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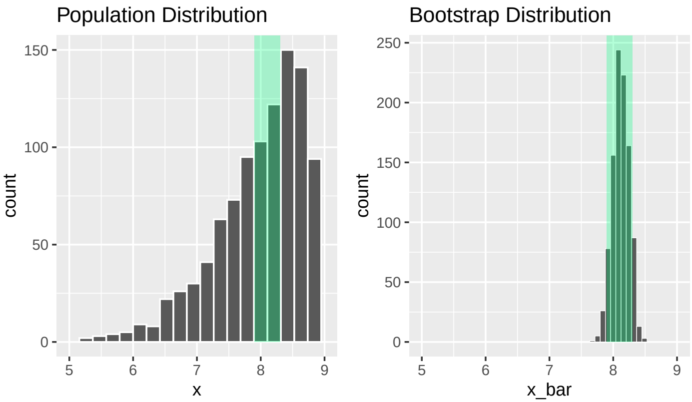

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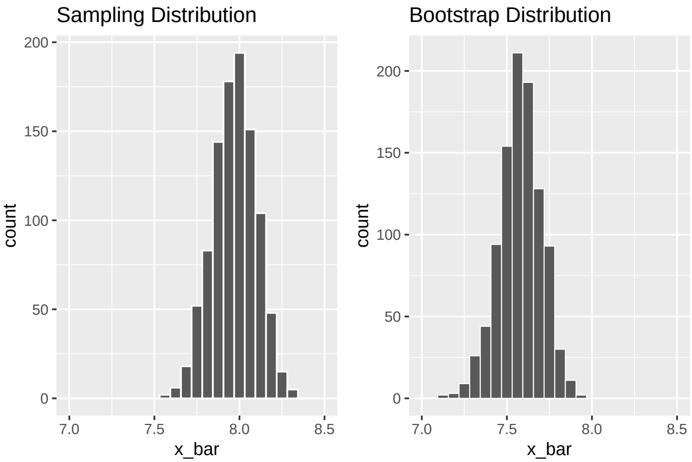

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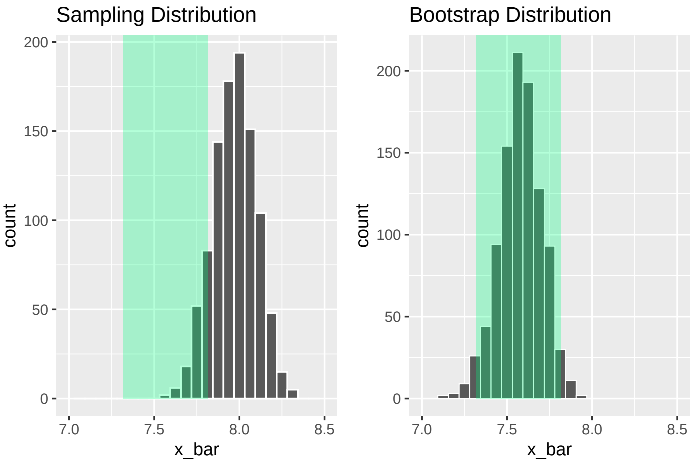

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  - One sample happened to have a sample mean of 4.9, producing a confidence interval of $(4.6, 5.2)$.
  - Based on what you know about sleep patterns, do you think there is a 95% chance this interval contains the true parameter?
  - What is a plausible alternative explanation for this interval?

Section 3

The `infer` package

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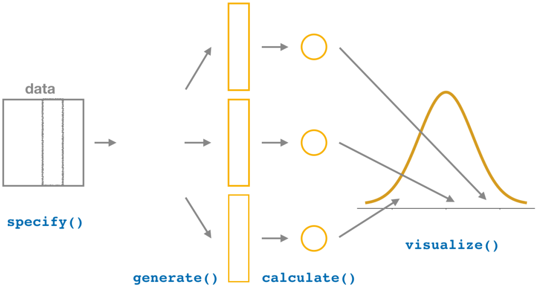

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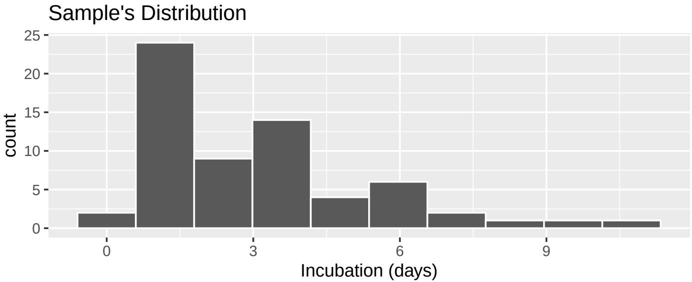

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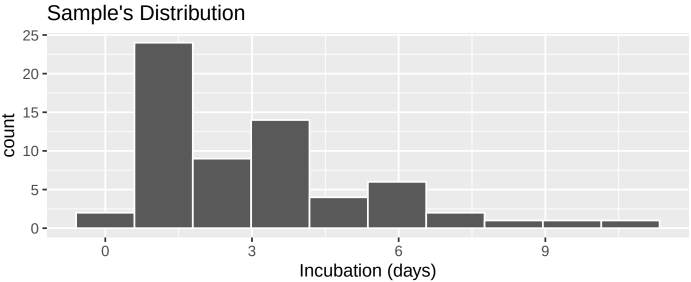

- What is the population of interest? What is the parameter?
- What is the sample? What is the statistic?

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- To investigate the infection rate

```
covid %>%
  specify(response = Incubation)
```

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- The resulting data frame has a number of rows equal reps × sample_size

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- After applying `calculate` the resulting data frame consists of one bootstrap statistic for each replicate (saved to the variable `stat`)

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- By using specify and calculate (and omitting generate) we can do just that, paralleling similar calculation for the bootstrap statistics

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covid_stat<- covid %>%
  specify(response = Incubation) %>%
  calculate(stat = "mean")
covid_stat
```

```
## # A tibble: 1 x 1
##    stat
##   <dbl>
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- Note: we saved the value of this calculation as covid_stat so we could use it later

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```
covid_boot<- covid %>%
  specify(response = Incubation) %>%
  generate( reps = 2000, type = "bootstrap") %>%
  calculate(stat = "mean")

head(covid_boot)
```

```
## # A tibble: 6 x 2
##   replicate  stat
##       <int> <dbl>
## 1         1  2.65
## 2         2  3.15
## 3         3  2.67
## 4         4  3.39
## 5         5  3.27
## 6         6  3.35
```

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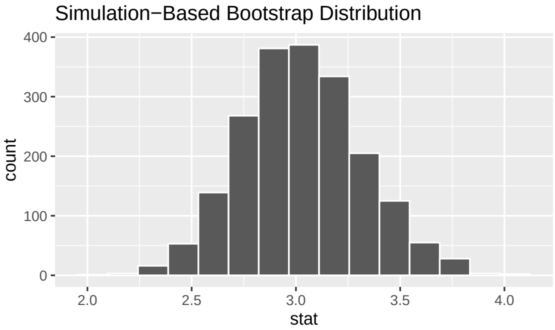

get_confidence_interval to... Get Confidence Interval

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```
percentile_ci<-covid_boot %>%
  get_ci(level = .95, type = "percentile")
percentile_ci
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1     2.49     3.63
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## 1     2.49     3.63
```
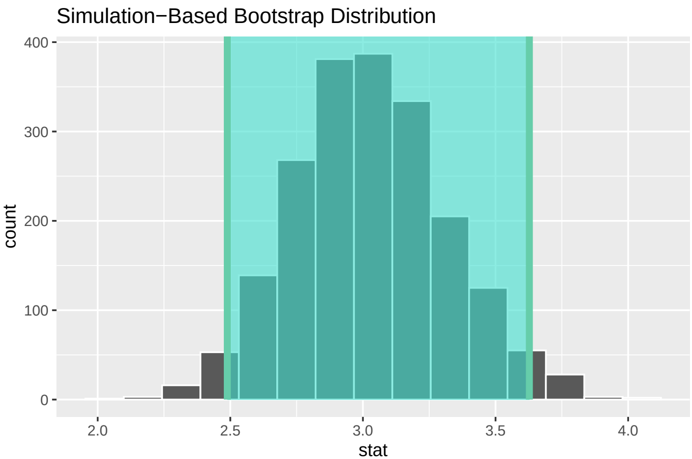

- When using the percentile type, the first value printed is the lower and the second is the upper bound.

Shade Confidence Intervals

- Once you've used get_ci to obtain endpoints of the confidence interval, you can shade the sampling distribution with the confidence interval region.

```
covid_boot %>% visualize()+shade_ci(endpoints = percentile_ci)
```



Standard Error Method

- The confidence interval using the standard error method will be of the form

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```
se_ci<-covid_boot %>%
  get_ci(level = .95, type = "se", point_estimate = covid_stat)
se_ci
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
## 1     2.46     3.60
```

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se_ci
```

```
## # A tibble: 1 x 2
##   lower_ci upper_ci
##      <dbl>    <dbl>
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- Note: for the se method, we also need to specify our point estimate (which is why we saved it as a variable before)

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- Why?

visualize Confidence Intervals

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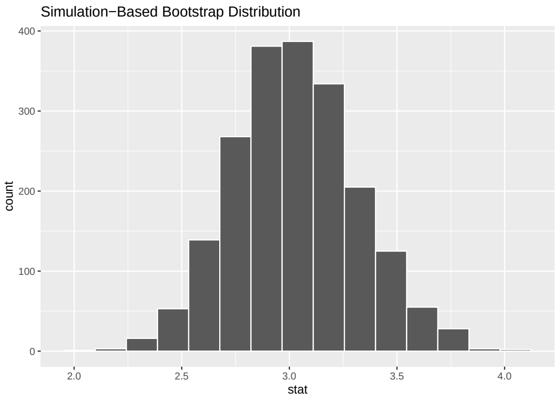
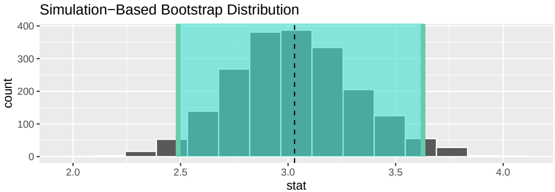
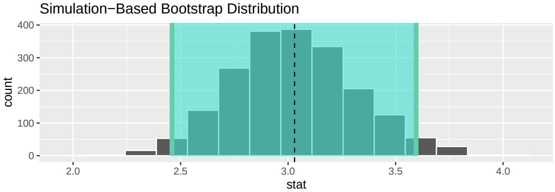


visualize Confidence Intervals

```
covid_boot %>% visualize() +
  shade_confidence_interval(endpoints = percentile_ci)+
  geom_vline(xintercept = 3.03, linetype = "dashed")
```

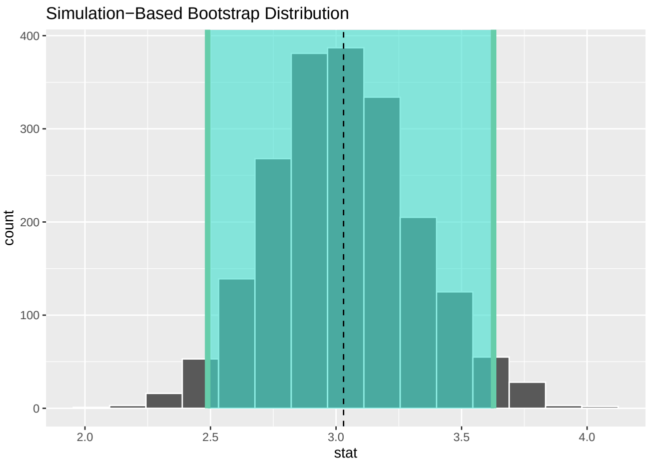



```
covid_boot %>% visualize() +
  shade_confidence_interval(endpoints = se_ci)+
  geom_vline(xintercept = 3.03, linetype = "dashed")
```



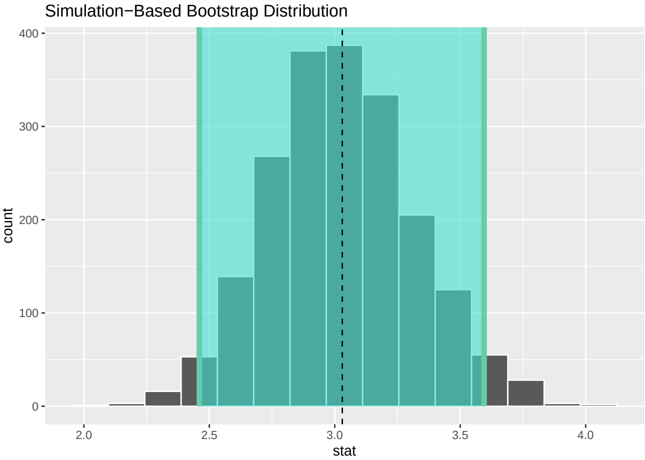
visualize Confidence Intervals

Percentile Method



visualize Confidence Intervals

SE Method



visualize Confidence Intervals

SE Method (with Percentile in blue)