Probability

Nate Wells

Math 141, 3/28/22

Outline

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- Introduce key definitions for probability theory
- Define conditional probability
- Discuss the Law of Total Probability and Tree Diagrams

Section 1

Probability Theory

Random Processes and Events

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    - Sometimes, statisticians distinguish between the words **outcome** and **event**, where *outcome* is used to refer to the minimal observable result of a process, while event refers to a collection of outcomes.
    - We won't be overly concerned about this distinction, and refer to all results as *events* (This distinction is further explored in Math 113 and Math 391)

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- Since probabilities are defined in terms of proportions, they will always be values between 0 and 1.
- For brevity, we'll represent statements like *the probability of the event "the coin lands heads" is 50%* using the notation:

$$P(\text{the coin lands heads}) = 0.5 \quad \text{or} \quad P(\text{Heads}) = 0.5 \quad \text{or} \quad P(H) = 0.5$$

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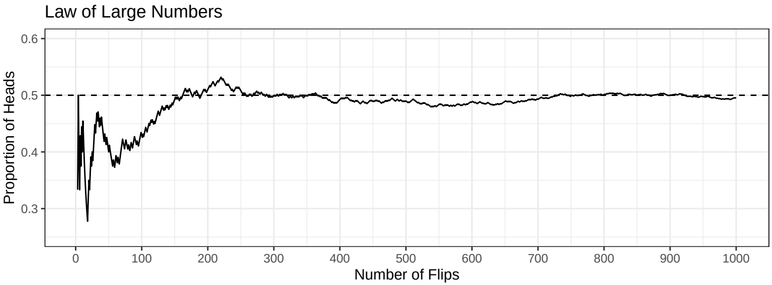

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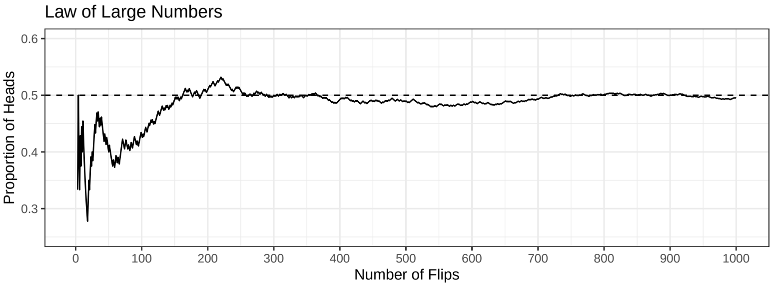

- Note that the proportion of heads deviates (significantly) from 0.5 during the first 50 flips, but stabilizes around 0.5 by 1000 flips.

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- Whenever we discuss probability, we are always (either explicitly or implicitly) defining a probability model.
- A large component of statistical inference is comparing observed data to the results we would expect under a certain probability model.

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$$P(\text{ starts with f }) = P(\text{ roll a 4 or roll a 5 }) = P(\text{ roll a 4 }) + P(\text{ roll a 5 }) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

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    - What is a certain event for this experiment?

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$$P(\text{ roll something other than a 1 }) = 1 - P(\text{ roll a 1}) = 1 - \frac{1}{6} = \frac{5}{6}$$

Section 2

Conditional Probability

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3. What is the probability that a random student was a sophomore who preferred coffee?
4. What is the probability that a random sophomore preferred coffee?

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- Find the probability that a randomly chosen student is a sophomore who likes coffee.

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The Law of Total Probability

One useful trick for computing probabilities is the following:

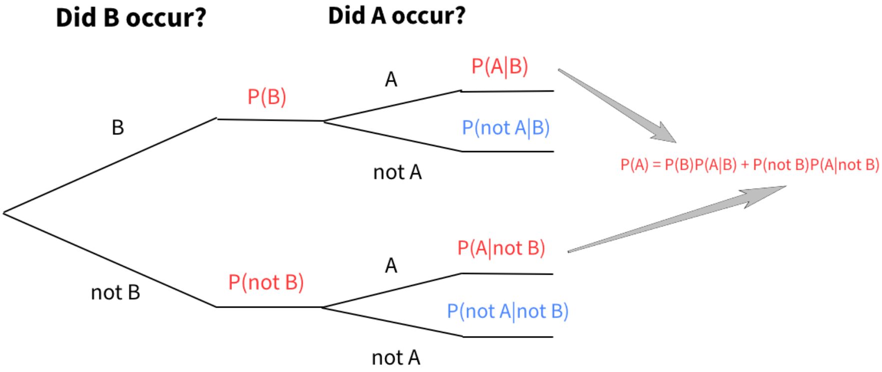
Theorem (The Law of Total Probability)

Let A and B be events. Then

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

- We can often represent the Law of Total Probability using a Tree Diagram:

Tree Diagrams



Lost Marbles

Two boxes contain a different number of red and blue marbles. The first box contains 20% red marbles while the second contains 80% red marbles. Suppose we select a marble from box 1 25% of the time and a marble from box 2 75% of the time. **What is the probability that a red marble is selected?**

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- Under what circumstances will $P(A|B)$ be much larger than $P(B|A)$? Much smaller?

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$$P(A|B) = P(B|A)\frac{P(A)}{P(B)}$$

- Why is this rule true?
- Under what circumstances will $P(A|B) = P(B|A)$?
- Under what circumstances will $P(A|B)$ be much larger than $P(B|A)$? Much smaller?
- Suppose $P(B|A) = 1$.
    - What does this suggest about A and B?

Bayes' Rule

To relate $P(A|B)$ and $P(B|A)$, we use the following theorem:

Theorem (Bayes' Rule)

Let A and B be events. Then

$$P(A|B) = P(B|A)\frac{P(A)}{P(B)}$$

- Why is this rule true?
- Under what circumstances will $P(A|B) = P(B|A)$?
- Under what circumstances will $P(A|B)$ be much larger than $P(B|A)$? Much smaller?
- Suppose $P(B|A) = 1$.
  - What does this suggest about A and B?
  - What is $P(A|B)$ in this case?